# PATENT APPLICATION

of

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for

# Solid Catadioptric Lens with a Single Viewpoint

## FIELD OF THE INVENTION

The present invention relates generally to lenses for applications such as vision and imaging where a single viewpoint is required, and in particular to catadioptric lenses having a single viewpoint.

## BACKGROUND OF THE INVENTION

Applications in computational vision such as ego-motion estimation and tracking often require imaging a large field of view. It is also desirable for an imaging system to have a single viewpoint in order to produce geometrically correct perspective images from the panoramic images obtained from the imaging system. The single and fixed viewpoint constraint requires that an imaging system only capture light that is passing through a single point in three dimensional

space, thus sampling the 5-dimensional plenoptic function at that point, which is known as the effective viewpoint. Such imaging systems have been realized by assemblies of curved mirrors and lenses. Although these catadioptric systems can have a large field of view and can satisfy the single viewpoint requirement they are cumbersome, expensive and large.

A number of wide field of view optics use facing mirrors to produce high quality images at the expense of an obscuration of the field of view. A simple example of a prior art optical system **10** of such type is shown in Fig. 1. System **10** has two facing mirrors **12, 14** aligned on an optical axis **16.** Mirror **14** receives and reflects light from a scene **20** to mirror **12.** Mirror **12** reflects the light back to mirror **14** such that the light passes a central aperture **18** in mirror **14** and projects an image **22** of scene **20** on a screen **24.**

Mirror **12** obscures a cone-shaped central portion **26** of the field of view seen by mirror **14.** The result of this obscuration is a shadow **28** in image **22.** To be imaged, light arriving from scene **20** has to be incident on mirror **14** at an angle larger than the angle of cone **26,** e.g., at an angle of incidence $\theta_i$ relative to axis **16.**

The prior art contains teachings about telescopes and other systems that employ the above-described principles. For example, U.S. Pat. No. 5,089,910 to Sigler teaches a catadioptric zoom relay telescope with two mirrors of which

the first mirror or primary mirror is aspheric. U.S. Pat. No. 5,940,222 to Sinclair et al. teaches catadioptric zoom lens assemblies employing these same principles.

Some applications such as robot vision and panoramic imaging require the optical system to have a single point of view or viewpoint. This condition enables the optical system to produce a perspective view of objects in its field of view. In some applications, it is further desirable to maximize the size of the field of view imaged from the single viewpoint.

It is difficult to satisfy all of the above conditions with a single system. The catadioptric zoom relay telescopes and lens assemblies are usually limited to a small field of view and many do not have a single viewpoint. In fact, many high quality systems offer excellent on-axis performance, e.g., as taught by Hicks in U.S. Pat. No. 6,412,961 but are inherently not single viewpoint. U.S. Pat. No. 5,473,474 to Powell teaches a panoramic lens that images a large field of view but lacks a single viewpoint. Still another approach using the facing mirrors configuration is taught by Kuroda et al. in U.S. Pat. No. 5,854,713. This patent describes a reflection type angle of view transforming optical apparatus that is hollow but also lacks a single viewpoint. Of further interest is the catadioptric system of Charles shown in U.S. Pat. No. 6,449,103. Additional references of interest include U.S. Pat. No. 4,566,763 teaching the use of a parabolic reflector and U.S. Application No. 2003/0142203 teaching the use of a hyperbolic reflector.

The prior art does teach some systems that have a single viewpoint. For example, U.S. Pat. No. 3,505,465 to Rees discloses how to use a hyperbolic mirror to produce a single viewpoint system for a person viewing an image such as a TV or video image produced, e.g., by a video camera. More recently, Patent Nos. 5,760,826 and 6,118,474 to Nayar describe an imaging apparatus for imaging a scene with a substantially paraboloid-shaped reflector whose focus is coincident with the single viewpoint. The imaging apparatus has a telecentric means optically coupled to the paraboloid-shaped reflector for filtering out principal rays of electromagnetic radiation that are not orthographically reflected by the paraboloid-shaped reflector.

Unfortunately, none of the prior art teachings provide a compact, effective and easy to manufacture single viewpoint lens that can be used in vision systems requiring a panoramic projection or perspective view of objects distributed over a large field of view.

**OJECTS AND ADVANTAGES**

In view of the above, it is an object of the invention to provide a compact, effective, rugged and easy to manufacture catadioptric lens with a single viewpoint and a large field of view. In particular, it is an object of the invention to provide a single viewpoint catadioptric lens that can be used in vision systems such as vision systems requiring a panoramic projection.

These and other objects and advantages of the invention will become apparent upon reading the ensuing description.

## SUMMARY OF THE INVENTION

The objects and advantages of the invention are secured by a solid catadioptric lens with a single point of view or viewpoint. The lens has an optical axis and a spherical refractive surface with a center C on the optical axis. The lens has an ellipsoidal reflective surface facing the spherical refractive surface arranged such that a first focus $F_1$ of the ellipsoidal reflective surface is coincident with the center C of the spherical refractive surface. The single viewpoint of the lens is at the center C of the spherical reflective surface. Further, the lens has a shaping surface facing the ellipsoidal reflective surface for shaping a light that passes the single viewpoint.

In one set of embodiments the shaping surface is a refractive shaping surface and the lens has an aperture for enforcing the single viewpoint at center C. The aperture is positioned at a second focus $F_2$ of the ellipsoidal reflective surface. The lens is constructed such that the second focus $F_2$ of the ellipsoidal reflective surface is on the optical axis, like first focus $F_1$.

In some embodiments where the shaping surface is a refractive shaping surface, the second focus $F_2$ is located near or even on the refractive shaping surface. In some other embodiments

using a refractive shaping surface, the second focus $F_2$ is inside the lens. The refractive shaping surface can assume a variety of shapes, but it is preferably ellipsoidal, thus forming an ellipsoidal refractive shaping surface. For light shaping reasons, it is also preferred that the ellipsoidal refractive shaping surface have its first focus $F_1'$ coincident with the second focus $F_2$ of the ellipsoidal reflective surface. In some embodiments, it is also convenient that the ellipsoidal refractive shaping surface have a conic constant $K_2$ equal to a conic constant $K_1$ of the ellipsoidal reflective surface.

In another set of embodiments the shaping surface is a reflective surface and an aperture is used for enforcing the single viewpoint. For example, in some embodiments in this set the aperture is positioned on the ellipsoidal reflective surface. In alternative embodiments, the aperture is positioned beyond the ellipsoidal reflective surface. Again, for light shaping reasons the reflective surface is preferably a second ellipsoidal reflective shaping surface. Furthermore, in some specific embodiments the second ellipsoidal reflective shaping surface has a first focus $F_1'$ coincident with the second focus $F_2$ of the ellipsoidal reflective surface. In some of these specific embodiments the second ellipsoidal reflective shaping surface has a conic constant $K_2$ equal to the conic constant $K_1$ of the ellipsoidal reflective surface.

It should be noted that whether the shaping surface is refractive or reflective it can assume various shapes including conic sections. Alternatively, the shaping surface can be flat. Furthermore, the shaping surface does not have to be only refractive or reflective; it can be semi-transparent.

The solid catadioptric lens is preferably made of a optical material with an index n. Suitable materials include glasses, plastics and other well-known optical materials.

The invention further provides for a single viewpoint vision system. The vision system employs the solid catadioptric lens for shaping light passing through the single viewpoint. It should be noted that the vision system can be used for projecting light or for collecting light for functions such as scanning or imaging, respectively. In embodiments where the lens is used for imaging, an imaging element can be provided for imaging the light onto an imaging unit or screen in an imaging plane. In scanning applications a scanning arrangement is provided.

In some embodiments an optical relay is used for passing the light from the lens to the application. For example, an optical relay can be used to pass the light to an imaging plane when the lens is deployed in an imaging vision system. In scanning vision systems, an optical relay can be used to deliver light from a scan element, e.g., a scan mirror to the lens.

The details of the invention will now be explained in the detailed description with reference to the attached drawing figures.

## BRIEF DESCRIPTION OF THE DRAWINGS

Fig. 1 (Prior art) is a three-dimensional diagram illustrating a class of off-axis optical systems.

Fig. 2 is a cross-sectional side view of one embodiment of a lens according to the invention.

Fig. 3 is a cross-sectional side view of the lens of Fig. 2 used in imaging.

Fig. 4 is a cross-sectional side view illustrating another embodiment of a lens according to the invention.

Fig. 5 is a cross-sectional side view of a lens according to the invention with folded geometry.

Fig. 6 is a cross-sectional side view of another lens with folded geometry.

Fig. 7 is a cross-sectional side view of a folded lens used for projecting or imaging.

Fig. 8 is a three-dimensional diagram showing a vision system having an imaging element and employing the lens of Fig. 7.

Fig. 9 is cross-sectional side view showing another vision system having a scanning arrangement and employing the lens of Fig. 7.

Fig. 10 is a cross-sectional side view of yet another lens according to the invention.

Fig. 11 is a cross-sectional side view of the lens similar to the one shown in Fig. 2 with an optical relay.

## DETAILED DESCRIPTION

The present invention will be best understood by first referring to an embodiment of a single viewpoint solid catadioptric lens **30** as shown in Fig. 2 in cross-sectional side view. Lens **30** has an optical axis **32** and is made of an optical material of index n. Preferably, the optical material is uniform and exhibits substantially no or very little variation of index n. Suitable materials include glasses and plastics, such as moldable plastics and other optical materials.

Lens **30** has a spherical refractive surface **34** with a center C on optical axis **32**. A light **36** propagating to and a light **36'** propagating from center C traverses spherical refractive surface **34** at normal incidence. Spherical refractive surface **34** defines a solid angle Θ over which lens **30** gathers light **36** and into which lens **30** projects light **36'**.

Lens **30** has an ellipsoidal reflective surface **38** facing spherical refractive surface **34**. Ellipsoidal reflective surface **38** can be made reflective by providing it with a reflective coating or film **40**, as shown in this embodiment, or by other means. Ellipsoidal reflective surface **38** is created by revolving a mathematical ellipse, as shown in dashed lines **38'**. Specifically, surface **38** is defined by revolving ellipse **38'** about optical axis **32**. Surface **38** is

oriented such that a first focus $F_1$ is coincident with center C of spherical refractive surface **34** on optical axis **32.** Second focus $F_2$ defined by surface **38** is also on optical axis **32** and within lens **30**.

Lens **30** has a shaping surface **42** facing ellipsoidal reflective surface **38** for shaping light **36, 36'** passing through center C. In fact, center C is the point of view or viewpoint of lens **30**. An aperture **44** is provided to ensure that center C is the only viewpoint of lens **30**. In other words, aperture **44** enforces the single viewpoint of lens **30**. Preferably, aperture **44** is formed by an object embedded in the optical material of lens **30** and having a pin-hole or diaphragm at second focus $F_2$. Alternatively, aperture **44** is defined in a non-transparent section of the optical material of lens **30**. A person skilled in the art will appreciate that there are many alternative ways of defining aperture **44**.

Shaping surface **42** can assume any shape and be located at any position past or even at aperture **44,** as found suitable for shaping light **36, 36'** for a given application. In the present embodiment shaping surface **42** is a refractive shaping surface for passing light **36, 36'** in and out of lens **30**. In fact, refractive shaping surface **42** is an ellipsoidal refractive shaping surface formed by revolving a mathematical ellipse **42'**, as shown in dashed lines, about optical axis **32**. For light shaping reasons, it is preferred that ellipsoidal refractive shaping surface **42** have its first focus $F_1'$ coincident with second focus $F_2$ defined by surface **38,** and its

second focus $F_2'$ on optical axis **32** within lens **30.** Even more preferably, surface **42** has a conic constant $K_2$ equal to:

$$K_2 = -\frac{1}{n^2},$$

where n is the index of the optical material. Under these circumstances, light **36** entering lens **30** over solid angle $\Theta$ emerges from lens **30** through surface **42** in a direction substantially parallel to optical axis **32.** It is further possible to set a conic constant $K_1$ of surface **38** equal to conic constant $K_2$, as in the present embodiment.

A plate **46** is placed before lens **30.** When lens **30** collects light **36** arriving at an angle of incidence $\theta_i$ to optical axis **32,** then plate **46** can be a screen for projecting light **36** thereon. In this mode lens **30** can be used for imaging. Alternatively, plate **46** emits light **36'** substantially parallel to optical axis **32** into lens **30.** In this mode lens **30** projects light **36'** into solid angle $\Theta$ and can be used for projecting an image.

The operation of lens **30** for imaging a wide field of view is explained with reference to the cross-sectional side view of Fig. 3. The field of view is defined by solid angle $\Theta$ over which lens **30** gathers light **36.** Note that angle $\Theta$ is symmetric about optical axis **32.**

Lens **30** is positioned such that its single viewpoint at center C is at a height $h$ above an object plane **48**. A number of object points $P_1$, $P_2$, $P_3$, $P_4$ are located on object plane **48** (points $P_3$ and $P_4$ are far away from optical axis **32** on plane **48**). Light **36** emanates from object plane **48** and propagates in ray bundles **50, 52, 54** and **56** to lens **30.** Point $P_1$ lies at one edge of the field of view such that ray bundle **50** emanating from it enters lens **30** at minimum incidence angle $\theta_{min.}$ Point $P_4$ lies at the other edge of the field of view such that ray bundle **56** emanating from it enters lens **30** at maximum incidence angle $\theta_{max.}$ The design of lens **30** enables maximum incidence angle $\theta_{max.}$ to be nearly 90°. Ray bundles **50** and **56** are drawn in dashed lines to indicate that they bound the field of view of lens **30.**

Ray bundles **50,** … **56** enter lens **30** via spherical refractive surface **34.** They then pass through the single point of view at center C and propagate to reflective ellipsoidal surface **38.** At surface **38** ray bundles **50,** … **56** are reflected to ellipsoidal refractive shaping surface **42.** Aperture **44** enforces the single point of view by allowing ray bundles **50,** … **56** to continue propagating to ellipsoidal refractive shaping surface **42** while stopping light that enters via spherical refractive surface **34** but does not pass through the single point of view at center C.

Since conic constants $K_1$ and $K_2$ of surfaces **38** and **42** are equal, and $K_1=K_2=-1/n^2$, and foci $F_2$ and $F_1'$ coincide, light **36** of ray bundles **50,** … **56** is substantially parallel to optical

axis **32** upon exiting lens **30**. This is a desirable type of shaping because ray bundles **50**, … **56** are projected onto image plane **46** without requiring further optical shaping. Points $P_1$, … $P_4$ on object plane **48** are thereby imaged to corresponding imaged points $P_1^I$, … $P_4^I$ on image plane **46**.

An advantageous feature of lens **30** is that ray bundles **50**, … **56** impinging on refractive surface **34** at equal angular intervals or field angles are mapped substantially linearly to distances from optical axis **32** when exiting ellipsoidal refractive shaping surface **42**. This means that object points $P_1$, … $P_4$ at equal field angles are mapped to imaged points $P_1^I$, … $P_4^I$ at substantially equal separations from each other in image plane **46**. The substantially linear map between field angle and distance from optical axis in image plane **46** is useful in many imaging applications and is sometimes referred to as f-θ imaging.

Fig. 4 is a cross-sectional side view of another embodiment of a single viewpoint catadioptric lens **60**. Lens **60** is made of a moldable plastic of index n and has a spherical refractive surface **62** with a center C on an optical axis **64**. Surface **62** faces an ellipsoidal reflective surface **66**. The two surfaces are arranged such that a first focus $F_1$ defined by surface **66** coincides with center C. A shaping surface **68** faces surface **66**. In this embodiment shaping surface **68** is reflective and is parabolic-shaped, thus forming a paraboloid reflective shaping surface. Surface **68** is defined by a single focus $F_1'$ lying on optical axis **64**. Moreover, focus

$F_1'$ coincides with a second focus $F_2$ of surface **66.** In this embodiment conic constants $K_1$ and $K_2$ of surfaces **66** and **68** are not equal.

Lens **60** has an aperture **70** for enforcing the single viewpoint at center C. Since light cannot pass through paraboloid reflective shaping surface **68,** aperture is positioned at facing surface **66.** In the present embodiment aperture **70** is defined in or on ellipsoidal reflective surface **66.** For example, aperture **70** is a pin-hole in a reflective coating or film covering surface **66.** Alternatively, aperture **70** can be placed beyond surface **66** or before it.

A projecting unit **72** for emitting light **36'** is positioned behind surface **66** and centered on optical axis **64.** Unit **72** can be an array of light emitters and is used herein to illustrate the application of lens **60** for projecting light **36'** into the field of view indicated by angle $\Theta$. The field of view defines a minimum emission angle $\sigma_{min.}$ and a maximum emission angle $\sigma_{max.}$. Unit **72** emits light **36'** from its emitters such as a pixel **74** far enough from optical axis **64** to map to an emission angle $\sigma$ such that $\sigma_{min.}<\sigma<\sigma_{max.}$. For example, light **36'** emitted from a pixel **74** at a distance d to optical axis **64,** is admitted into lens **60** through aperture **70** and emitted at emission angle $\sigma$. A person skilled in the art will recognize that additional optical elements such as lenses and mirrors can be placed between surface **66** and unit **72** for performing various light shaping and guiding functions as may be required for any particular application.

In one application, lens **60** is used for projecting light **36'** into its field of view. Unit **72** activates pixel **74** for emitting light **36'** at emission angle $\sigma$ to optical axis **64**. Light **36'** propagates parallel and offset by distance d to optical axis **64** and is admitted into lens **60** via aperture **70**. The light guiding properties of lens **60** map light **36'** to angle $\sigma$ as it exits lens **60** through spherical refractive surface **62**.

In another application lens **60** is used for collecting light **36** arriving from the field of view, e.g., at an angle of incidence $\theta_i=\sigma$, where angle of incidence $\theta_i$ is larger than a minimum incidence angle $\theta_{min.}=\sigma_{min.}$ and smaller than a maximum incidence angle $\theta_{min.}=\sigma_{max.}$.

The geometry of the single viewpoint catadioptric lens of the invention can be modified in many ways. For example, the ellipsoidal surfaces of the lens can be folded. Fig. 5 illustrates an embodiment of a folded single viewpoint catadioptric lens **80** having external profiles **82** and **84**. Lens **80** has a spherical refractive surface **86** with a center C coincident with the single viewpoint. An ellipsoidal reflective surface **88** having a first focus $F_1$ and a second focus $F_2$ faces surface **86** and its first focus $F_1$ is coincident with center C. A shaping surface **90**, here in the form of a second reflective ellipsoidal shaping surface having first and second foci $F_1'$, $F_2'$ faces surface **86**. All foci are on an optical axis **87**.

Ellipses **92, 94** that are revolved to form surfaces **88** and **90** are indicated in dashed lines for clarification. In contrast to previous embodiments, ellipse **94** has its second focus $F_2'$ on the other side of the viewpoint or center C and even beyond surface **88**. The overlap of ellipses **92, 94** and surfaces **88, 90** created by their revolution produces a geometry that is herein referred to as folded.

Second reflective ellipsoidal shaping surface **90** is formed on external profile **82** with the aid of a reflective coating or film (not shown). The size of surface **90** can decreased to enlarge the field of view of lens **80**. External profile **84** extends from surface **88** and terminates at a flat coupling face **96**. Face **96** is transparent and located such that second focus $F_2'$ of surface **90** falls on it. In fact, the size of face **96** defines the aperture of lens **80** that enforces the single viewpoint at center C. If a small aperture is desired, then a mask can be supplied on face **96** and a central opening in the mask can define the aperture.

To understand the operation of lens **80** we follow an incident ray bundle **98** of light **36** entering lens **80**, while realizing that lens **80** can also project light **36'**. Light **36** enters lens **80** through surface **86**, passes single viewpoint at center C and is reflected by surface **88**. Light **36** then propagates to second focus $F_2$ as mathematics dictate that light passing through one focus of an ellipsoid and internally reflected converge at its second focus. By design, second focus $F_2$ is

coincident with first focus $F_1'$ of surface **90** that is also ellipsoidal and thus enforces the same mathematical rule on light **36**. Specifically, light **36** is reflected from surface **90** and converges at second focus $F_2'$ on coupling face **96**.

Light **36** is out-coupled through face **96** and may be further shaped, deflected and/or steered as required by the deployment conditions of lens **80**. For example, imaging elements can be positioned past face **96** to guide light **36** into an image plane. Alternatively, optical relays can be positioned there to shape and guide light **36**.

Another folded geometry is embodied by a catadioptric lens **100** shown with its ray-trace in Fig. 6. The basic geometry of lens **100** is described by nine geometric design parameters: R, $R_1$, $K_1$, $R_2$, $K_2$, $L_1$, $L_2$, $L_3$, and $L_4$. The first five of these represent: radius of spherical refractive surface **102**, radius of curvature of ellipsoidal reflective surface **104**, conic constant of surface **104**, radius of curvature of second ellipsoidal reflective shaping surface **106**, conic constant of surface **106**. Parameters $L_1$, $L_2$, $L_3$, and $L_4$ correspond to lengths as shown. Additionally, **108** is an apex of surface **106**, **110** is an apex of surface **104**, and **112** is an aperture enforcing the singe viewpoint of lens **100**. Note that second focus $F_2'$ of surface **106** falls on surface **104** within aperture **112**. Note that surface **106** is on an external profile **118** in this embodiment.

The design of lens **100** is a complete closed form solution. Thus, the optical performance of any vision system using lens **100** mostly depends on the design of its optical relay. In the present embodiment a single relay lens **114** represents an optical relay for producing images at a single wavelength of light **36** in an image plane **116**. A more achromatic optical relay with a field flattener can be used for color imaging, as will be appreciated by a person skilled in the art.

Lens **100** can be scaled to different sizes by changing the values of R, $R_1$ and $R_2$. The general shape of lens **100** and its angular magnification is controlled by the ratio of the parameters $K_1$ and $K_2$. Once these parameters have been adjusted, then the following equations are used to find $L_1$, $L_2$, $L_3$ and $L_4$:

$$L_1 = R_1 \frac{\left[1-\sqrt{-K_1}\right]}{1+K_1};$$

$$L_2 = \frac{2R_1\sqrt{-K_1}}{1+K_1};$$

$$L_3 = \frac{R_2\left[1-\sqrt{-K_2}\right]}{1+K_2};$$

$$L_4 = \frac{2R_2\sqrt{-K_2}}{1+K_2}.$$

This ensures that the first geometrical focus $F_1'$ of second ellipsoidal reflective shaping surface **106** is coincident with second geometrical focus $F_2$ of ellipsoidal reflective surface **104**. The present embodiment illustrates a special case where

$L_4 = L_1 + L_2$, superposes second geometrical focus $F_2'$ of second ellipsoidal reflective shaping surface **106** on apex **110** of ellipsoidal reflective surface **104**. As noted above, this causes aperture **112**, which may have a diameter of 2 mm or even less, depending on the desired light intensity, to fall on apex **110**. Preferably, aperture **112** is just a portion of ellipsoidal reflective surface **104** that is masked off before coating entire surface **104** with a reflective coating such as aluminum.

Fig. 7 illustrates still another folded single viewpoint lens **120** without any external profiles and a field of view described by solid angle $\Theta$. Lens **120** has a spherical refractive surface **122** facing an ellipsoidal reflective surface **124** such that a center C of surface **122** is coincident with first focus $F_1$ of surface **124**. Surface **124** faces a semi-transparent ellipsoidal shaping surface **126** such that a second focus $F_2$ of surface **124** is coincident with a first focus $F_1'$ of surface **126**. Second focus $F_2'$ of surface **126** falls on surface **124** within an aperture **128**. Center C and all foci are on an optical axis **130** of lens **120**.

An optical relay **132** is positioned next to surface **124** for guiding light **36'** generated by a display unit **134** to lens **120**. More precisely, relay **132** is a telecentric relay for guiding light **36'** that propagates parallel to optical axis **130** into lens **120**. Relay **132** converts a distance d at which light **36'** is offset from optical axis **130** to an admission angle $\alpha$ into lens **120**. Reference f indicates a focal length

of relay **132**. By virtue of the design of relay **132**, admission angle $\alpha$ is a substantially linear function of distance d over small values of angle $\alpha$.

During operation, a pixel **136** of display unit **134** offset by distance d from optical axis **130** generates light **36'**. Relay **132** guides light **36'** into lens **120** at admission angle $\alpha$. Light **36'** is reflected from semi-transparent surface **126**, surface **124** and then exits lens **120** at an emission angle $\sigma$ through surface **122**. During this process angle $\alpha$ is "amplified" to angle $\sigma$ and the angular magnification is given by the ratio $\sigma/\alpha$. A portion **138** of light **36'** exits lens **120** via semi-transparent surface **126**. Portion **138** can be used for monitoring the operation of lens **120** and/or display unit **134**. Portion **138** can also be used for verify alignment of lens **120** and display unit **134** or other functions. A skilled artisan will appreciate that the degree of transparency of semi-transparent surface **126** can be adjusted depending on these functions. It should also be noted that the transparency of semi-transparent surface **126** can depend on wavelength of light **36'**.

In an alternative embodiment, lens **120** operates in reverse by admitting light **36** and projecting it to pixel **136**. In this case display unit **134** is replaced by a light-sensing element, e.g., an imaging element such as an imaging array. In such array pixel **136** is light-sensitive. Note that a portion of light **36** is also transmitted through semi-transparent surface **126**.

From the small sample of designs described above, it is clear that catadioptric lenses according to the invention are versatile, simple, rugged and easy to manufacture. They can be made in parts or as a whole part by techniques such as molding. Their optical performance depends largely on the accuracy of the surfaces and the design of auxiliary optical relays that deliver light to or collect light from them. Such relays should be designed based on the vision system or application and specific design choices may depend on whether it is a system with an imaging element, a scanning arrangement or a display element.

Fig. 8 illustrates a single viewpoint vision system **140** using lens **120** with a reflective ellipsoidal shaping surface **126'** rather than semi-transparent shaping surface **126**. Vision system **140** has an imaging element **142**, e.g., an imaging array with pixels **144** (shown partially for clarity). Array **142** is positioned in an image plane **146** behind surface **124**. The optical relay between surface **124** and image plane **146** is not shown here for reasons of clarity. However, any type of relay, including a telecentric relay as described in Fig. 7, can be used between lens **120** and imaging array **142**.

Surface **126'** limits the field of view of lens **120** to the off-axis region described by solid angle $\Theta$. Therefore, vision system **140** is most useful when the center of the field of view is obscured or does not require imaging. The center of the field of view corresponds to shadow **148** in image plane

**146**. For clarity of explanation, aperture **128** for enforcing point of view C is not indicated on surface **124** and image plane **146** is shown enlarged.

Vision system **140** is directed at an arbitrary object **150** to be imaged. Herein, object **150** is a substrate whose surface lies in an X′-Y′ plane of object coordinates (X′,Y′,Z′). Vision system **140** is above substrate **150** and the origin of object coordinates and single viewpoint C of lens **120** are connected by a vector $R^c$. Vector $R^c$ is collinear with optical axis **130** and the norm of vector $R^c$ is the distance between the origin and viewpoint C. Vision system **140** moves in space and its coordinates (X,Y,Z) are rotated with respect to object coordinates (X′,Y′,Z′). Many conventions exist to align the object coordinates and vision system coordinates. For example, the rotation between vision system coordinates (X,Y,Z) and object coordinates can be expressed by three consecutive rotations by Euler angles $(\varphi,\theta,\psi)$.

During operation vision system **140** collects light **36**, such as light **36** from a point P on substrate **150**. Light **36** propagates in a ray bundle **152** to enter lens **120** at angle of incidence $\theta_i$. Lens **120** images point P to an imaged point $P^I$ on imaging array **142** in image plane **146**. Conveniently, image plane **146** is described by imaging array axes $X^I$ and $Y^I$, and the location of imaged point $P^I$ in image plane **146** is described by a vector $R^P$. Because of its single viewpoint C, lens **120** produces a panoramic projection of imaged point $P^I$.

Thus, lens **120** can produce a panoramic image of object **150** in image plane **146**.

In a modified embodiment, vision system **140** can use lens **120** to project light **36′** to a projected point $P_p$ on substrate **150** and collect light **36** from point P simultaneously. In this embodiment a pixel **154** at point $P^S$ on array **142** produces light **36′**. The location of point $P^S$ in image plane **146** is described by vector $R^S$. A ray bundle **156** of light **36′** is projected through lens **120** at emission angle $\sigma$ onto substrate **150** to point $P_p$. Hybrid arrays having light sensitive and light-emitting pixels that can be employed in this embodiment are known in the art. Of course, a display unit such as the one shown in Fig. 7 can be used for projecting light **36′** if no imaging of substrate **150** is required.

Fig. 9 shows a vision system **160** employing lens **120** with a wavelength-dependent semi-transparent ellipsoidal shaping surface **126″** to project light **36′** on an object **162**. At certain points object **162** responds by producing a back-scattered light **36″** and system **160** simultaneously uses lens **120** to collect back-scattered light **36″**.

System **160** has a display unit **164**, e.g., a display screen or a detector array, in front of surface **126″**. Unit **164** is centered on optical axis **130**. Further, system **160** has a scanning arrangement **166** with a light source **168** for generating light **36′**. Arrangement **166** has a scan mirror **170** for deflecting or steering light **36′**. Mirror **170** is tilted

by a scan angle $\gamma$ with respect to a mirror plane $M_P$. Drivers for controlling scan angle $\gamma$ are well-known in the art.

Arrangement **166** has an optical relay **172** for shaping and directing light **36'** along optical axis **130** into lens **120** through aperture **128.** Relay **172** is of the type that converts scan angle $\gamma$ to a corresponding admission angle $\alpha$ into lens **120.** For example, relay **172** is a 4-f system using two lenses with the same or different focal lengths on axis **130** such that the center of mirror **170** is coincident with one focal point of relay **172** and second focus $F_2'$ of surface **126"** is coincident with another focal point of the 4-f system. Relay **172** also has a beam splitter **176** for deflecting any light returning to scanning arrangement **166** from lens **120.**

In operation, system **160** directs light **36'** by adjusting scan angle $\gamma$ of mirror **170.** A change in scan angle $\gamma$ changes emission angle $\sigma$ at which light **36'** exits lens **120.** In the present case scan angle $\gamma$ is varied such that light **36'** is first emitted at angle $\sigma_1$ and then at $\sigma_2$.

The pass wavelength of semi-transparent surface **126"** is chosen such that a small portion **174** of light **36'**, e.g., a fraction of a percent, is transmitted and projected onto display unit **164.** Specifically, when light **36'** is emitted at angle $\sigma_1$ portion **174** is transmitted to point $P_2^I$. Then, at angle $\sigma_2$ portion **174** is transmitted to point $P_1^I$. Portion **174** of light **36'** can be used for reference, feedback, tracking or other auxiliary functions.

At angle $\sigma_2$ object **162** produces back-scattered light **36″** that returns to lens **120** along the path of light **36′**. Light **36″** enters lens **120** through surface **122** at an angle of incidence $\theta_i=\sigma_2$. A small fraction of light **36″** exits lens **120** via surface **126″**. The remainder of light **36″** is reflected by surface **126″** and is separated by beam-splitter **174**. Of course, if the fraction of light **36″** registered by unit **164** is sufficient for monitoring back-scattered light **36″** then the remainder of light **36″** can be discarded. In those cases beam-splitter **174** can be removed.

Fig. 10 illustrates yet another embodiment of a single viewpoint lens **180**. Lens **180** has a spherical refractive surface **182** facing an ellipsoidal reflective surface **184**, which in turn faces a flat refractive shaping surface **186**. First focus $F_1$ of surface **184** and center C of surface **182** are coincident at the single viewpoint of lens **180**. The second focus $F_2$ of surface **184** is on surface **186** and within aperture **188** enforcing the single viewpoint.

Lens **180** is made of two materials **183**, **185** having indices $n_1$, $n_2$ respectively. Material **183** is preferably a glass or plastic, while material **185** is a glass or plastic, but could also be a liquid or optical gel filling a cavity in lens **180**. In the latter case, a portion **190** of lens **180** indicated in dashed and dotted lines is made of a suitable material that can also be a glass or plastic forming an envelope to contain material **185**. It is preferable that $n_1=n_2$.

An optical relay in the form of a compound lens **192** is positioned adjacent to surface **186** for out-coupling light **36** from lens **180**. In the present case lens **192** is designed for projecting light **36** onto an image plane **194**. In reverse, lens **192** can be used for in-coupling light **36'** from image plane **194** into lens **180**.

Alternatively, lens **180** can be built with the aid of a shell **196** indicated in dashed lines and defining an internal cavity **198**. Cavity **198** is filled, e.g., with an optical gel or other optical material. In fact, entire lens **180** including portion **190** can be a hollow form or shell filled with an optical material such as an optical gel or liquid.

Fig. 11 is a cross-sectional side view of a lens **200** similar to lens **30** shown in Fig. 2 and equipped with an optical relay **210**. Lens **200** has a spherical refractive surface **202**, a facing ellipsoidal reflective surface **204** and a refractive ellipsoidal shaping surface **206**. First focus $F_1$ of surface **204** and center C of surface **202** are coincident at the single viewpoint. Foci $F_1'$ and $F_2$ are also coincident. All foci are on optical axis **208**.

Lens **200** differs from lens **30** in that it has no fixed aperture within its body. Rather, an adjustable aperture **212** is provided in optical relay **210** between lenses **214, 216**. Lenses **214** and **216** have focal lengths $f_1$ and $f_2$ respectively. Relay **210** has a first image plane **218** at a distance equal to

$f_1$ from lens **214**. A second image plane **220** is located at a distance equal to $f_2$ from lens **216**. A person skilled in the art will realize that relay **210** is a type of 4-f relay.

During operation aperture **212** can be adjusted to regulate the F-number of lens **200** and operate in a wide range of illumination conditions. It should be noted that opening aperture **212** to obtain a low F-number results in progressive deterioration of the single viewpoint property of lens **200**.

The single viewpoint catadioptric lens according to the invention admits of myriads of other embodiments. For example, it does not have to be a solid lens, but can also be hollow, e.g., in cases where it is made by molding. In some cases where certain regions of the lens are hollow cavities left by the fabrication method, these hollow cavities can be filled with an optical fluid or gel having an index of refraction matching that of the solid sections of the lens. In still other embodiments the ellipsoidal reflective surface and/or the shaping surface are counter-sunk within the lens rather than being on an external profile. Given all the alternative embodiments, the scope of the invention is to be judged by the following claims and their legal equivalents.